

Analytical Expressions for the Hard-Scattering Production of Massive Partons

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Abstract. We obtain explicit expressions for the two-particle differential cross section and the two-particle angular correlation function in the hard-scattering production of massive partons in order to exhibit the “ridge” structure on the away side in the hard-scattering process. The single-particle production cross section is also obtained and compared with the ALICE experimental data for charm production in pp collisions at 7 TeV at LHC.

1. Introduction

Knowledge of massive quark production processes in pp collisions provides useful insight to guide our intuition in heavy quark production in nucleus-nucleus collisions. Analytical expressions for these processes summarize important features and essential dependencies so as to facilitate the uncovering of dynamical effects wherever they may occur. Similar analyses in massless quark production have led to new insights in the dominance of the hard-scattering process over a large p_T domain and have paved the way for locating the boundary between the hard-scattering process and the flux-tube fragmentation process in high-energy pp collisions [1, 2].

Accordingly, we would like to obtain $E_c E_\kappa d\sigma(AB \rightarrow c\kappa X)/d\mathbf{c} d\mathbf{\kappa}$ for the two-particle differential cross section in the production of massive partons c and κ . From such a general result, we integrate out the transverse momenta and obtain the two-particle angular correlation function $d\sigma/d\Delta\phi d\Delta y$ where $\Delta\phi = \phi_\kappa - \phi_c$ and $\Delta y = y_\kappa - y_c$, exhibiting analytically the “ridge” structure on the away side at $\Delta\phi \sim \pm\pi$ in the hard-scattering process. We subsequently examine $d\sigma(AB \rightarrow cX)/dy_c c_T dc_T$ for the single-particle spectrum and compare with ALICE experimental data for charm production in pp collisions at 7 TeV at LHC [3].

2. Hard Scattering Integral for $E_c E_\kappa d\sigma(AB \rightarrow c\kappa X)/d\mathbf{c} d\mathbf{\kappa}$

In the parton model, the hard-scattering cross section for $AB \rightarrow c\kappa X$ is given by [4]

$$d\sigma(AB \rightarrow c\kappa X) = \sum_{ab} \int K_{ab} dx_a d\mathbf{a}_T dx_b d\mathbf{b}_T G_{a/A}(x_a, \mathbf{a}_T) G_{b/B}(x_b, \mathbf{b}_T) d\sigma(ab \rightarrow c\kappa), \quad (1)$$

where (x_a, \mathbf{a}_T) and (x_b, \mathbf{b}_T) represent the momenta and $G_{a/A}$ and $G_{b/B}$ the structure functions of the incident partons a and b respectively, and K_{ab} is the correction factor which can be obtained perturbatively [5] or it can also be approximated nonperturbatively [6]. The quantity $d\sigma(ab \rightarrow c\kappa)$ is the cross section element for the process $ab \rightarrow c\kappa$,

$$d\sigma(ab \rightarrow c\kappa) = \frac{1}{4[(a \cdot b)^2 - m_a^2 m_b^2]^{1/2}} |T_{fi}|^2 \frac{d^3 c}{(2\pi)^3 2E_c} \frac{d^3 \kappa}{(2\pi)^3 2E_\kappa} (2\pi)^4 \delta^4(a + b - c - \kappa). \quad (2)$$

Here, we normalize the Dirac fields by $\bar{u}u = 2m$. The quantity $|T_{fi}|^2$ is related to $d\sigma/dt$ by

$$|T_{fi}|^2 = 16\pi[\hat{s} - (m_a + m_b)^2][\hat{s} - (m_a - m_b)^2] \frac{d\sigma(ab \rightarrow c\kappa)}{dt}. \quad (3)$$

We consider the simplified case with $m_a = m_b = 0$ and treat a_T, b_T as small perturbations. The cross section element is then

$$d\sigma(ab \rightarrow c\kappa) = \frac{s_{ab} d\sigma(ab \rightarrow c\kappa)}{2\pi} \frac{d^3c}{E_c} \frac{d^3\kappa}{E_\kappa} \delta^4(a + b - c - \kappa), \quad (4)$$

where $\hat{s} = s_{ab} = (a + b)^2$ that is different from $s = s_{AB} = (A + B)^2$. We get

$$\frac{E_c E_\kappa d\sigma(AB \rightarrow c\kappa X)}{d^3c d^3\kappa} = \sum_{ab} \int K_{ab} dx_a d\mathbf{a}_T dx_b d\mathbf{b}_T G_{a/A}(x_a, \mathbf{a}_T) G_{b/B}(x_b, \mathbf{b}_T) \frac{\hat{s} d\sigma(ab \rightarrow c\kappa)}{2\pi} \frac{d\sigma}{dt} \delta^4(a + b - c - \kappa). \quad (5)$$

We consider a factorizable structure function with a Gaussian intrinsic transverse momentum distribution,

$$G_{a/A}(x_a, \mathbf{a}_T) = G_{a/A}(x_a) \frac{1}{2\pi\sigma^2} e^{-\mathbf{a}_T^2/2\sigma^2}. \quad (6)$$

Upon integrating over the transverse momenta \mathbf{a}_T and \mathbf{b}_T , we obtain

$$\begin{aligned} \frac{d\sigma(AB \rightarrow c\kappa X)}{dy_c dc_T d\phi_c dy_\kappa d\kappa_T d\phi_\kappa} &= \sum_{ab} \int K_{ab} dx_a dx_b G_{a/A}(x_a) G_{b/B}(x_b) \frac{e^{-\frac{(\mathbf{c}_T + \mathbf{\kappa}_T)^2}{4\sigma^2}}}{2(2\pi\sigma^2)} \frac{\hat{s}}{2\pi} \frac{d\sigma(ab \rightarrow c\kappa)}{dt} \\ &\times \delta(a_0 + b_0 - (c_0 + \kappa_0)) \delta(a_z + b_z - (c_z + \kappa_z)). \end{aligned} \quad (7)$$

To carry out the integration over x_a and x_b , we write out the momenta in the infinite momentum frame,

$$a = \left(x_a \frac{\sqrt{s}}{2} + \frac{a^2 + a_T^2}{2x_a \sqrt{s}}, \mathbf{a}_T, x_a \frac{\sqrt{s}}{2} - \frac{a^2 + a_T^2}{2x_a \sqrt{s}} \right), \quad (8)$$

$$b = \left(x_b \frac{\sqrt{s}}{2} + \frac{b^2 + b_T^2}{2x_b \sqrt{s}}, \mathbf{b}_T, -x_b \frac{\sqrt{s}}{2} + \frac{b^2 + b_T^2}{2x_b \sqrt{s}} \right), \quad (9)$$

$$c = \left(x_c \frac{\sqrt{s}}{2} + \frac{c^2 + c_T^2}{2x_c \sqrt{s}}, \mathbf{c}_T, x_c \frac{\sqrt{s}}{2} - \frac{c^2 + c_T^2}{2x_c \sqrt{s}} \right), \quad (10)$$

$$\kappa = \left(x_\kappa \frac{\sqrt{s}}{2} + \frac{\kappa^2 + \kappa_T^2}{2x_\kappa \sqrt{s}}, \mathbf{\kappa}_T, -x_\kappa \frac{\sqrt{s}}{2} + \frac{\kappa^2 + \kappa_T^2}{2x_\kappa \sqrt{s}} \right), \quad (11)$$

where x_c and x_κ can be represented by y_c and y_κ

$$x_c = \frac{m_{cT} e^{y_c}}{\sqrt{s}}, \quad x_\kappa = \frac{m_{\kappa T} e^{y_\kappa}}{\sqrt{s}}. \quad (12)$$

The two delta functions in Eq. (7) can be integrated to yield

$$\frac{d\sigma(AB \rightarrow cdX)}{dy_c dc_T d\phi_c dy_\kappa d\kappa_T d\phi_\kappa} = \sum_{ab} K_{ab} x_a G_{a/A}(x_a) x_b G_{b/B}(x_b) \frac{e^{-\frac{(\mathbf{c}_T + \mathbf{\kappa}_T)^2}{4\sigma^2}}}{2\pi(4\pi\sigma^2)} \frac{d\sigma(ab \rightarrow c\kappa)}{dt}, \quad (13)$$

$$\begin{aligned} \text{where} \quad x_a &= x_c + \frac{\kappa^2 + \kappa_T^2}{x_\kappa s} - \frac{b^2 + b_T^2}{x_b s} = \frac{m_{cT} e^{y_c}}{\sqrt{s}} + \frac{m_{\kappa T} e^{-y_\kappa}}{\sqrt{s}} - \frac{b^2 + b_T^2}{x_b s}, \\ x_b &= x_\kappa + \frac{c^2 + c_T^2}{x_c s} - \frac{a^2 + a_T^2}{x_a s} = \frac{m_{\kappa T} e^{y_\kappa}}{\sqrt{s}} + \frac{m_{cT} e^{-y_c}}{\sqrt{s}} - \frac{a^2 + a_T^2}{x_a s}. \end{aligned} \quad (14)$$

The above explicit formula gives the cross section for the production of c and κ , when the elementary cross section $d\sigma(ab \rightarrow c\kappa)/dt$ is given explicitly in terms of its depending variables.

3. The angular correlation $d\sigma(AB \rightarrow c\kappa X)/d\Delta\phi d\Delta y$

We can represent \mathbf{c} and $\mathbf{\kappa}$ by (y_c, ϕ_c) and $(y_c + \Delta y, \phi_c + \Delta\phi)$, respectively. After averaging over y_c and ϕ_c , and integrating over c_T, κ_T , the correlation function (13) from the process $ab \rightarrow c\kappa$ is

$$\frac{d\sigma(AB \rightarrow c\kappa X)}{d\Delta\phi d\Delta y} = K_{ab} x_a G_{a/A}(x_a) x_b G_{b/B}(x_b) \delta_\sigma(\Delta\phi), \quad (15)$$

$$\text{where } \delta_\sigma(\Delta\phi) = \frac{1}{(4\pi\sigma^2)} \int_0^\infty c_T dc_T \int_0^\infty \kappa_T d\kappa_T \exp\left\{-\frac{c_T^2 + 2c_T\kappa_T \cos \Delta\phi + \kappa_T^2}{4\sigma^2}\right\} \frac{d\sigma(ab \rightarrow c\kappa)}{dt}. \quad (16)$$

The above analytical expression assumes a simple form for $a=b$ and $c=\kappa$. The structure function can be represented in the form $x_a G_{a/A}(x_a) \propto (1-x_a)^{g_a}$ for which the two-particle angular correlation function becomes

$$\frac{d\sigma(AB \rightarrow c\kappa X)}{d\Delta\phi d\Delta y} \sim A \left[1 - \frac{2m_{cT}}{\sqrt{s}} [\cosh y_c + \cosh(y_c + \Delta y)] + 2\left(\frac{m_{cT}}{\sqrt{s}}\right)^2 [1 + \cosh(2y_c + \Delta y)] \right]^{g_a} \delta_\sigma(\Delta\phi). \quad (17)$$

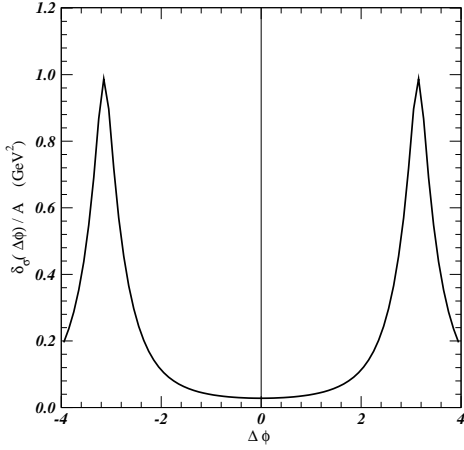


Fig. 1 The correlation function $\delta_\sigma(\Delta\phi)/A$

If we consider $d\sigma(ab \rightarrow c\kappa)/dt$ to be approximately of the form

$$\frac{d\sigma(ab \rightarrow c\kappa)}{dt} = \frac{A}{(1 + c_T^2/m_c^2)^{n/2}}, \quad (18)$$

where $n=4$ from pQCD, then the integration over c_T and κ_T in Eq. (16) gives $\delta_\sigma(\Delta\phi)/A$ as shown in Fig. 1. The correlation function has maxima at $\Delta\phi \sim \pm\pi$ and a minimum at $\Delta\phi=0$. It is relatively flat in Δy because $m_{cT}/\sqrt{s} \ll 1$. This gives the distribution in the form of a ridge structure on the away side at $\Delta\phi \sim \pm\pi$.

4. Production of massive quarks by gluons

We consider the process $gg \rightarrow c\bar{c}$, where analytical expressions $d\sigma/dt$ have been obtained earlier by Cambridge [7] and Glück, Owen, and Reya [8]. In the notation of [8], the cross section is

$$\frac{d\sigma(gg \rightarrow c\bar{c})}{dt} = \frac{\pi\alpha_s^2}{64s^2} \left[12M_{ss} + \frac{16}{3}M_{tt} + \frac{16}{3}M_{uu} + 6M_{st} + 6M_{su} - \frac{2}{3}M_{tu} \right]. \quad (19)$$

Upon writing out the above quantity as a function of $c_T, \bar{c}_T, \bar{y} = (y_c + y_{\bar{c}})/2$ and $\Delta y = y_{\bar{c}} - y_c$, we get the heavy-quark pair production cross section

$$\frac{d\sigma(AB \rightarrow c\bar{c}X)}{dy_c c_T dc_T d\phi_c dy_{\bar{c}} \bar{c}_T d\bar{\phi}_c} \sim AK_{ab}(1-x_a)^{g_a}(1-x_b)^{g_b} \frac{e^{-\frac{(\mathbf{c}_T + \bar{\mathbf{c}}_T)^2}{4\sigma^2}}}{2\pi(4\pi\sigma^2)} \frac{d\sigma(gg \rightarrow c\bar{c})}{dt}, \quad (20)$$

$$\text{where } \frac{d\sigma(gg \rightarrow c\bar{c})}{dt} = \frac{\pi\alpha_s^2}{4^5 m_{cT}^4 \cosh^4 \bar{y}} \left\{ \left[\frac{12}{\cosh^2 \bar{y}} + \frac{64}{3} \cosh 2\bar{y} - 24 \right] + \left(\frac{m_c}{m_{cT}} \right)^2 \left[\frac{64}{3} + \frac{24 \sinh^2 \bar{y}}{\cosh^2 \bar{y}} - \frac{8}{3} \right] \right. \\ \left. + \frac{m_c^4}{m_{cT}^4} \left[-\frac{64}{3} \frac{\cosh 2\bar{y}}{\cosh^2 \bar{y}} + \frac{8}{3} \frac{1}{\cosh^2 \bar{y}} \right] \right\},$$

$x_a = 2m_{cT} \cosh \bar{y} e^{\Delta y/2}$, and $x_b = 2m_{cT} \cosh \bar{y} e^{-\Delta y/2}$. This shows the back-to-back correlation of \mathbf{c}_T and $\bar{\mathbf{c}}_T$ and the relatively flat distribution as a function of Δy .

5. Single-particle charm production

We need to integrate over κ in Eq. (5) to get the single-particle distribution of c . Neglecting intrinsic p_T and integrating over κ , we get

$$\frac{E_c d\sigma(AB \rightarrow cX)}{d^3c} = \sum_{ab} \int dx_a dx_b G_{a/A}(x_a) G_{b/B}(x_b) \frac{\hat{s}}{\pi} \frac{d\sigma(ab \rightarrow cd)}{dt} \delta(\hat{s} + \hat{t} + \hat{u} - m_c^2 - m_{\bar{c}}^2). \quad (21)$$

The integral over x_a and x_b can be evaluated by the saddle point method [1], and we get for $\bar{y} = y_c \approx 0$,

$$\begin{aligned} \frac{E_c d\sigma(AB \rightarrow cX)}{d^3c} &\propto K_{ab} (1-x_a)^{g_a+1/2} (1-x_b)^{g_b+1/2} \frac{1}{\sqrt{x_c}} \frac{d\sigma(gg \rightarrow c\bar{c})}{dt} \\ &= \frac{AK_{ab}\alpha_s^2}{m_{cT}^{4.5} \cosh^4 \bar{y}} \left\{ \left[\frac{12}{\cosh^2 \bar{y}} + \frac{64}{3} \cosh 2\bar{y} - 24 \right] + \frac{m_c^2}{m_{cT}^2} \left[\frac{64}{3} + \frac{24 \sinh^2 \bar{y}}{\cosh^2 \bar{y}} - \frac{8}{3} \right] + \frac{m_c^4}{m_{cT}^4} \left[-\frac{64}{3} \frac{\cosh 2\bar{y}}{\cosh^2 \bar{y}} + \frac{8}{3} \frac{1}{\cosh^2 \bar{y}} \right] \right\}, \end{aligned} \quad (22)$$

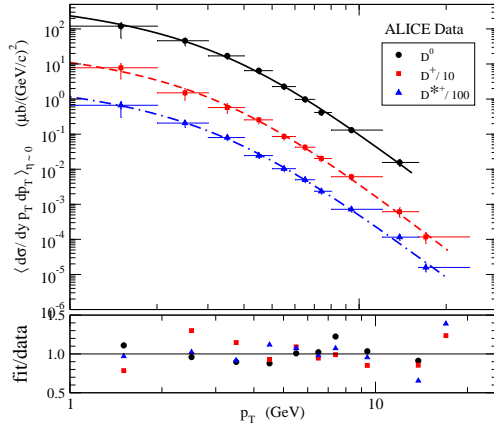


Fig. 2 . ALICE data for charm production.

The extracted values of n and m_0 are greater than those from the expected lowest-order results of $n = 4.5$ and $m_c \sim 1.5$ GeV in Eq. (22). This may arise from the final-state interactions correction factor K_{ab} [6] for production of the $c\bar{c}$ pair in the color singlet state. The attractive color-singlet interaction between c and \bar{c} enhances the production of the pair at lower p_T and increases the value of the effective mass of the produced charm meson.

6. Conclusion

We present analytical expressions for the hard-scattering production of massive quarks in order to guide our intuition, point out essential dependencies, and summarize important features. They will facilitate future comparisons with experimental data and pave the way for a better understanding of particle production processes.

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which contains $1/m_{cT}^4$, m_c^2/m_{cT}^6 , and m_c^4/m_{cT}^8 . We examine the ALICE charm production cross section data in pp collisions at 7 TeV [3] shown in Fig. 2. If we parametrize the data at mid-rapidity as $d\sigma/dy_c d\eta_T d\eta_T|_{\eta=0} \sim a/(1 + p_T^2/m_0^2)^{n/2}$, we find that the ALICE data can be fitted by a set of parameters given by

$d\sigma/dy dp_T d\eta_T _{\eta=0} = a/(1 + p_T^2/m_0^2)^{n/2}$			
Data	a [$\mu\text{b}/(\text{GeV}/c)^{-2}$]	n	m_0 (GeV)
D^0	1600	5.8	3.5
D^+	780	5.9	3.5
D^{*+}	808	5.7	3.5